Analysis of an Equivalent CCPE Connection Diagram of the One-port Circuit by Square-wave Voltage

Plamen Ivanov Nikovski and Ivan Mihaylov Maslinkov

Abstract - **The possibilities of using the phase demodulation are examined in order to determine the parameters of one-port circuits containing phase-constant elements in square-wave voltage. The results can be useful for the purpose of creating new or improving the performance of existing measured transmitters.**

Keywords – **Phase demodulation, Phase-constant element, Measured transmitter**

I. Introduction

Usually the parameters of one-port circuit (OP) operating in sine wave voltage are determined following phase quadrature demodulation of the current flowing through it [4]. The use of this approach in measuring converters, which steady state regime is non sine wave, can be of interest in practice. The study will dwell upon a case in which to a presented with an $CCPE$ equivalent scheme OP is applied square-wave voltage Fig.1. $CCPE$ is one-port circuit equivalent scheme composed of connected in parallel a ideal (perfect) capacitor and a phase-constant element. $CCPE$ OP is a generalization of the classical parallel RC OP in which the resistor is replaced with a phase-constant element described by differential equation of the following type:

$$A._{-\infty}D_t^{\alpha}u(t) = i_Y(t) \quad , \qquad (1)$$

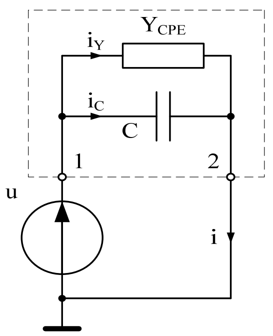

Fig.1. ***CCPE*** *one-port circuit*

where ${}_{-\infty}D_t^{\alpha}u(t)$ - is differentiation fractional operator of order α determined in accordance with the definition of Riemann-Liouville [2, 5], $A, \alpha \in \Re$, $0 \le \alpha \le 1$, $u = u(t)$ and $i_Y = i_Y(t)$ - the voltage of the leads of the phase-constant element and the current flowing through it respectively. Easily can be concluded that parallel RC OP is a special case of $CCPE$, when $\alpha = 0$.

Pl. Nikovski, Dep. Electrical Engineering and Electronics, University of Food Technologies - Plovdiv, 26 Maritza blvd., 4000 Plovdiv, Bulgaria, e-mail: plmnn@uft-plovdiv.bg

Iv. Maslinkov, Dep. Electrical Engineering and Electronics, University of Food Technologies - Plovdiv, 26 Maritza blvd., 4000 Plovdiv, Bulgaria, e-mail: imm@uft-plovdiv.bg

II. Presentation of ${}_{-\infty}D_t^{\alpha}u(t)$ in Frequency Domain

When we analyze the performance of electrical circuits described by fractional differential equations it is preferable to consider the relation between spectral characteristics of a given function $u(t)$ and its derivative ${}_{-\infty}D_t^{\alpha}u(t)$. Let us assume that $u(t)$ has spectral characteristics $U(j.\omega)$

$$u(t) \xleftrightarrow{F} U(j.\omega) \quad . \qquad (2)$$

Then [4]

$${}_{-\infty}D_t^{\alpha}u(t) \xleftrightarrow{F} (j.\omega)^{\alpha}.U(j.\omega) \quad . \qquad (3)$$

According to (3) fractional differentiation in time domain corresponds to multiplication by $(j.\omega)^{\alpha}$ in frequency domain.

III. Determination of the Average Current through the Elements of $CCPE$ OP

When $CCPE$ OP is powered by an ideal voltage source $u(t)$ Fig.1, the current $i(t)$ flowing through it carries charge:

$$q(t) = q_C(t) + q_Y(t) \quad , \qquad (4)$$

where

$$q_C(t) = C.u(t) \quad \text{and} \qquad (5)$$

$$\frac{dq_Y}{dt} = A.D^{\alpha}u(t) \quad . \qquad (6)$$

To calculate $q_Y(t)$ it is necessary to solve the differential equation (6). In case of periodic input signal that can be most easily done by using the Fourier transformation: equation (6) is represented in frequency domain

$$j.\omega.Q_Y(j.\omega) = A.(j.\omega)^{\alpha}.U(j.\omega) \quad ,$$

the spectral-response function is calculated $Q_Y(j.\omega)$

$$Q_Y(j.\omega)=\frac{A}{(j.\omega)^{1-\alpha}}.U(j.\omega) \quad \text{or}$$

$$Q_Y(j.\omega)=\frac{A}{\omega^{1-\alpha}}.e^{-j.(1-\alpha).\frac{\pi}{2}}.U(j.\omega) \qquad (7)$$

and finally the function we are examining

$$q_Y(t)\xleftarrow{F}Q_Y(j.\omega) \quad . \qquad (8)$$

Equations (4), (5) and (8) give a full understanding of the circulation and distribution of the charge in $CCPE$ OP.

The voltage $u(t)$ is square-wave signal with peak amplitude U_a and period T

$$u(t)=U_a.\mathrm{sgn}(\sin(\omega.t)) \quad , \qquad (9)$$

where $\omega=2.\pi/T$

which is represented by the following series of Fourier [3]:

$$u(t)=U_a.\frac{4}{\pi}.\sum_{k=1}^{\infty}\frac{1}{2.k-1}.\sin(2.k-1).\omega.t \quad . \qquad (10)$$

From (5) and (9) we receive

$$q_C(t)=C.U_a.\mathrm{sgn}(\sin(\omega.t)) \quad . \qquad (11)$$

Considering (7), (10) and the observation made in [1] we can determine

$$q_Y(t)=AU_a\left(\frac{1}{\omega}\right)^{1-\alpha}\frac{4}{\pi}\sum_{k=1}^{\infty}\frac{1}{(2k-1)^{2-\alpha}}\sin\left((2k-1)\omega t-(1-\alpha)\frac{\pi}{2}\right) \qquad (12)$$

The resulting convergence series represent periodic symmetric conjugation function: $q_Y(t)=-q_Y(t+T/2)$. Same properties characterize (11): $q_C(t)=-q_C(t+T/2)$. In this case the average current specified by the charge conveyed through the capacitor, phase-constant element and $CCPE$ OP in interval $\{[t_1+nT], [t_1+(2n+1)T/2]\}$, $n=0, \pm1, \pm2, \ldots$ will be respectively:

$$I_{avC}(t_1)=2.\frac{q_C(t_1+(2.n+1).T/2)-q_C(t_1+n.T)}{T}=$$

$$=2.\frac{q_C(t_1+T/2)-q_C(t_1)}{T}=-4.\frac{q_C(t_1)}{T} \quad \text{,or}$$

$$I_{avC}(t_1)=-\frac{4.C.U_a}{T}.\mathrm{sgn}(\sin(\omega.t_1)) \quad ; \qquad (13)$$

$$I_{avY}(t_1)=-\frac{4AU_a}{T}\left(\frac{1}{\omega}\right)^{1-\alpha}\frac{4}{\pi}\sum_{k=1}^{\infty}\frac{1}{(2k-1)^{2-\alpha}}\sin\left((2k-1)\omega t_1-(1-\alpha)\frac{\pi}{2}\right) \qquad (14)$$

$$I_{av}(t_1)=I_{avC}(t_1)+I_{avY}(t_1) \quad . \qquad (15)$$

Considering (13) and (14) as well as standard values

$$I_C(t_1)=\frac{T}{4.C.U_a}.I_{avC}(t_1)=-\mathrm{sgn}(\sin(\omega.t_1)) \qquad (16)$$

$$I_Y(t_1)=\frac{T}{4.A.U_a}.\left(\frac{1}{\omega}\right)^{1-\alpha}.I_{avY}(t_1) \quad \text{,or}$$

$$I_Y(t_1)=-\frac{4}{\pi}\sum_{k=1}^{\infty}\frac{1}{(2k-1)^{2-\alpha}}\sin\left((2k-1)\omega t_1-(1-\alpha)\frac{\pi}{2}\right) \qquad (17)$$

Fig.2 represents the relation $I_C=I_C(t_1)$. Fig.3 and Fig.4 represent the relation $I_Y=I_Y(t_1)$ applying different values of α.

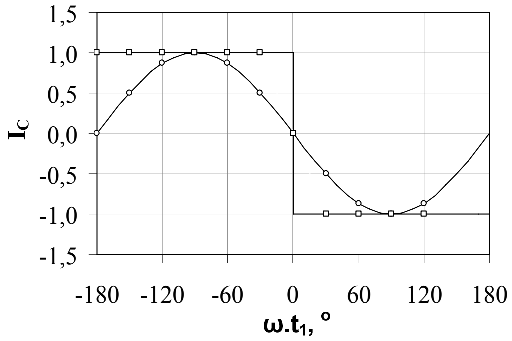

*Fig.2 Relation of the average current flowing through the capacitor from the starting point **t_1**:*
—o— with sine wave voltage;
—□— with square-wave voltage.

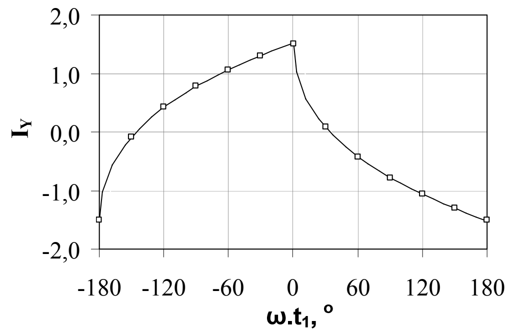

*Fig.3 Relation of the average current flowing through the phase-constant element from the starting point **t_1** with square-wave voltage and **α=0,5***

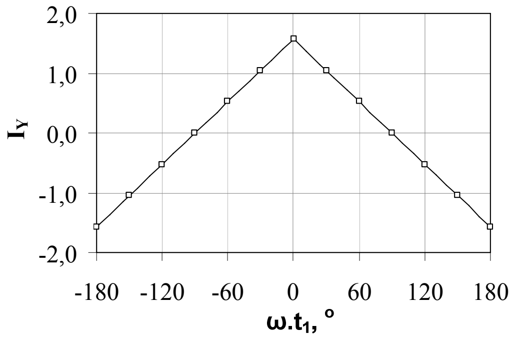

*Fig.4 Relation of the average current flowing through the phase-constant element from the starting point **t_1** with square-wave voltage and **α=0***

IV. DEFINING THE PARAMETERS OF *CCPE* OP

Assuming the following after a phase demodulation of $i(t)$ with reference signal in quadrature and phase with $u(t)$ the average value of the current flowing through $CCPE$ OP is achieved:

$$I_{av}(-T/4)=\frac{4CU_a}{T}+\frac{4AU_a}{T}\left(\frac{1}{\omega}\right)^{1-\alpha}\frac{4}{\pi}\sin\left(\alpha\frac{\pi}{2}\right)\sum_{k=1}^{\infty}\frac{(-1)^{k-1}}{(2k-1)^{2-\alpha}} \quad (18)$$

and

$$I_{av}(0_-)=\frac{4CU_a}{T}-\frac{4AU_a}{T}\left(\frac{1}{\omega}\right)^{1-\alpha}\frac{4}{\pi}\cos\left(\alpha\frac{\pi}{2}\right)\sum_{k=1}^{\infty}\frac{1}{(2k-1)^{2-\alpha}} \quad (19)$$

A and *C* can be calculated by consequently resolving the equations (18) and (19) having in mind the succession α of phase-constant element. As a result capacity *C* equals to

$$C=\frac{\pi}{4.U_a.\omega}.\left[(1-a).I_{av}(0_-)+(1+a).I_{av}(-T/4)\right] \quad , \quad (20)$$

where *a* equals to

$$a=\frac{\cos\left(\alpha\frac{\pi}{2}\right)\sum_{k=1}^{\infty}\frac{1}{(2k-1)^{2-\alpha}}+\sin\left(\alpha\frac{\pi}{2}\right)\sum_{k=1}^{\infty}\frac{(-1)^{k-1}}{(2k-1)^{2-\alpha}}}{\cos\left(\alpha\frac{\pi}{2}\right)\sum_{k=1}^{\infty}\frac{1}{(2k-1)^{2-\alpha}}-\sin\left(\alpha\frac{\pi}{2}\right)\sum_{k=1}^{\infty}\frac{(-1)^{k-1}}{(2k-1)^{2-\alpha}}} \quad (21)$$

and a is correlated only to α. As presented in Fig.5 this method of determining the capacity is appropriate when $\alpha < 0{,}6 \div 0{,}7$. In case of higher values of α growing rate of the coefficient a and its reliability towards α problems can arise for the practical realization of the method.

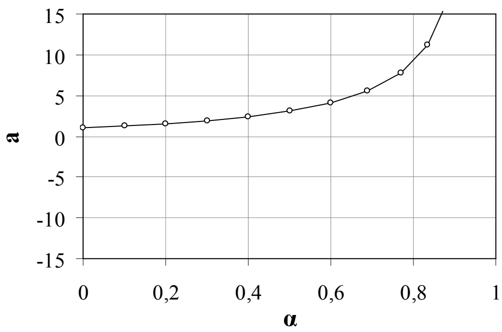

*Fig.5 Relation of **a** coefficient from the succession α of phase-constant element.*

V.CONCLUSION

Correlations (18)÷(21) indicate that it is possible to use phase demodulation in order to determine parameters of the OP presented with $CCPE$ equivalent circuit in a periodic square-wave signal. In a similar manner to the described above correlation can be discovered between other forms of input voltages. Regardless of the need to produce evidence attention is drawn to the fact that while average value of the current through the capacitor element C in sine wave voltage is

$$I_C(t_1)=\frac{T}{4.C.U_a}.I_{avC}(t_1)=-\sin(\omega.t_1) \quad . \quad (22)$$

Comparing (16) and (22) we conclude that in the square-wave voltage the average value of the current through the element *C* equals the highest value of the current reached under sine wave voltage and is not influenced by t_1 (respectively of phase $\omega.t_1$) of within every semi period (Fig.5). As a result the output signal of the measuring transmitters while using square-wave voltage will be less reliable to phase distortions from the input circuit or out of it.

REFERENCES

[1] Никовски, П. *Приложение на преобразованието на Fourier за изследване на линейни електрически вериги описани с фрактални диференциални уравнения*, Н. конф. с межд. участие „Хранителна наука, техника и технологии 2009", 23 - 24.10.2009 год., УХТ-Пловдив, Пловдив.

[2] Николов, Е. *Специални математически функции и фрактални оператори*, ТУ-София, София, 2004, 190с.

[3] Опенхайм, А., А. Уилски. *Сигнали и системи*, Техника, София, 1993, 661с.

[4] Стоянов, Ив. *Измерване в електрониката и изчислител-ната техника*, Техника София, 1987, 231с.

[5] Vinagre, B. *Fractional Calculus Fundamental*, 41st IEEE Conference on Decision and Control, 2002, 1-14p.